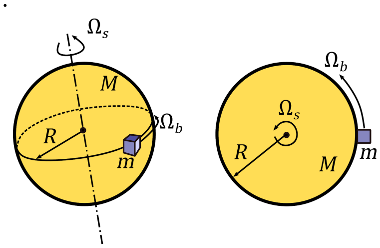
**SPIN CHANGE OF RUBBLE PILES DUE TO MASS SHEDDING CULMINATING FROM SURFACE MOTION.** D. Banik[1] and K. Gaurav[2], [1]Department of Aerospace Engineering, Indian Institute of Technology Kanpur (banik@iitk.ac.in), [2] Department of Mechanical Engineering, Indian Institute of Technology Kanpur.

**Introduction:** Previous studies on the spin change of rubble-pile asteroids have discussed the effect of exogenic events like meteoroid impacts and volatile release from pyrosilicate dehydration. The mathematical treatment of such events shows a finite change in spin rate [1, 2]. However, spin changes caused by mass shedding culminating from endogenic events like landslides over relatively undeformable cores have only been recently studied [3]. It has been shown that the surface motion of boulders is a greater contributor to spin change as compared to launch and crash events that occur as a result of the former [3]. In this direction, we shall present a simplified model, analytical solutions to which attest to the *negligible* spin change due to shedding ensued and preceded by surface motion in *contrast* to that caused by exogenic events.

**Problem formulation:** Imagine a sphere (or any axis-symmetric shape) of mass $M$ and radius $R$ that is to rotate about an axis passing through its center with a block of mass $m$ placed at the *equator*; see Fig. 1. Both of them are initially at rest. The block is being pulled towards the sphere and vice versa due to gravity. The sphere is suddenly given an *initial* angular velocity $\Omega_i$. Due to friction, the block gradually spins up and the sphere spins down. Let the *instantaneous* angular velocities of the sphere and the block be $\Omega_s$ and $\Omega_b$ respectively.




(a) (b)

**Figure 1: Schematic for the (a) isometric and (b) top-views of the problem description.**

**Equations of motion:** From the free-body diagram (FBD) of the block in the radial and tangential directions respectively we get,

$$N = \frac{GMm}{R^2} - m\Omega_b^{\,2}R \qquad (1)$$

and

$$\mu N = mR\frac{d\Omega_b}{dt}, \qquad (2)$$

where $N$ is the normal reaction force and $\mu$ is the coefficient of friction. Additionally, we calculate the *orbital* angular velocity of the block $\Omega_o$ for the radial location $R$ by setting $N$ to zero in Eq. 1.

At any time instant, from the conservation of angular momentum for the sphere-block system, we have the following,

$$I_s\Omega_i = I_s\,\Omega_s + I_b\Omega_b. \qquad (3)$$

where, $I_s$ and $I_b$ are the moment of inertia of the sphere and the block respectively.

**Steady-state solutions:** In steady-state i.e. when the spin rate of either body has ceased to change with time, $\Omega_s$ and $\Omega_b$ must be equal, else friction would still act to reduce the relative surface velocity between them. Let us call this the *steady-state* angular velocity $\Omega_{ss}$. Thus,

$$\Omega_{ss} = \frac{I_s\Omega_i}{I_s+I_b}. \qquad (4)$$

However, this will be true only when $\Omega_{ss} < \Omega_o$. This is because the block can gain an angular velocity of $\Omega_o$ at most as the normal reaction would vanish at $\Omega_{ss} = \Omega_o$. The maximum limit on $\Omega_i$ such that $\Omega_{ss} < \Omega_o$ is obtained by replacing $\Omega_{ss}$ by $\Omega_o$ in (4). This gives,

$$\Omega_i^{max} = \frac{(I_s+I_b)\Omega_o}{I_s}. \qquad (5)$$

Note that $\Omega_i^{max}$ is greater than $\Omega_o$ because the conservation of angular momentum spins the sphere down in addition to spinning the block up. For $\Omega_i >= \Omega_i^{max}$ the block would asymptotically gain *orbital* angular velocity $\Omega_o$ due to an asymptotic decrease of the normal reaction $N$ and consequently the frictional force to zero. This, in turn, prevents any further exchange of angular momentum between the sphere and the block with the latter slipping on the former. We find the *steady-state* angular velocity of the sphere $\Omega_{ss_s}$ from the balance of angular momentum.

$$\Omega_{ss_s} = \frac{I_s\Omega_i - I_b\Omega_o}{I_s}. \qquad (6)$$

**Evolution of spin rates:** Eliminating $N$ from (1) and (2) we get the differential equation for the evolution of $\Omega_b$. Note that, $\Omega_b$ is independent of $m$.

*Case 1.* $(\Omega_i < \Omega_i^{max})$ : Friction acts only until a relative surface velocity exists between the sphere and the block i.e. as long the system takes to reach steady-state. The time taken to reach steady-state is obtained on solving the ODE by integrating $\Omega_b$ from 0 to $\Omega_{ss}$ and $t$ from 0 to $T$. Therefore,

$$T = \frac{1}{\mu\Omega_o}\tanh^{-1}\left(\frac{\Omega_{ss}}{\Omega_o}\right). \qquad (7)$$

The time evolution of $\Omega_b$ is given by,

$$\Omega_b = \Omega_o\tanh(\mu\Omega_o t). \qquad (8)$$

Note that the evolution of $\Omega_b$ is independent of $\Omega_i$. However, $T$ is dependent on $\Omega_i$ through $\Omega_{ss}$. $\Omega_s$ is calculated from (3).

*Case 2.* $(\Omega_i >= \Omega_i^{max})$ : Since the domain of $\tanh^{-1}$ does not accommodate absolute values greater than one, the steady-state angular velocity of the block for all $\Omega_i >= \Omega_i^{max}$ will be $\Omega_o$. The time taken to reach steady-state will correspondingly be infinite. As $\Omega_b$

tends to $\Omega_o$ i.e. the orbital velocity, the normal reaction $N$ becomes negligibly small. Friction and hence the transfer of angular momentum, consequently reduce to asymptotically to zero. This happens irrespective of $\Omega_i$.

**Results and discussion:** Since we are looking at axisymmetric shapes it is worthwhile to discuss results with respect to rubble-pile asteroids that approximately satisfy the condition of axisymmetry; for example Bennu and Ryugu [4]. In a setting like Bennu, $R = 250\,\text{m}$, $M = 7.5 \times 10^{10}$ Kg, boulder size is $2\,\text{m}$ [5] and $m = 10^4$ Kg.

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|---|---|
| $\Omega_o$ | 2.0358 rad/hr |
| $T_o$ | 3.0836 hr |
| $\Omega_i^{max}$ | $\Omega_o + 6.2244 \times 10^{-7}$ rad/hr |

We shall consider different values of: the initial spin rate $\Omega_i$ for the sphere and the friction coefficient. Figure 2(a) shows the spin evolution of the sphere and the block for $\Omega_i = 1.62$ rad/hr. The effect of the surface motion on the spin rate of the sphere is negligible compared to the block because of the orders of magnitude difference in their masses. The effect of $\Omega_i$ on $\Omega_b$ is not seen in how the latter evolves but is reflected in termination criteria for evolution. In Fig. 2b, the time taken to reach steady-state is seen to increase with $\Omega_i$.

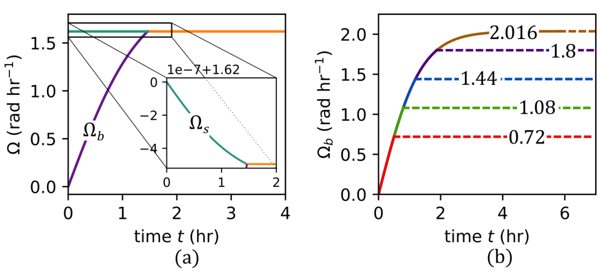


**Figure 2: (a) Evolution of $\Omega_s$ and $\Omega_b$ with time for $\Omega_i = 1.62$ rad/hr. (b) Evolution of $\Omega_b$ with time for different $\Omega_i$ (rad/hr). The friction coefficient for all cases is 0.364.**

The steady-state angular velocity is unaffected by the friction coefficient; see Fig. 3a . This is expected because $\Omega_{ss}$ comes directly from the conservation of angular momentum which does not contain $\mu$. The time required to reach steady state $T$ follows a rectangular hyperbolic relationship with $\mu$. The steady state is never reached if $\mu = 0$ and is reached immediately when $\mu$ tends to infinity. This can be inferred from in (7) and observed in Fig. 3b.

**Conclusions:** In general, there are two separate contributions of three-dimensional surface motion on the angular acceleration of the core: *first*, the rate of change of angular momentum of the regolith with respect to the core, and *second,* the rate of change of the moment of inertia of the regolith. Because we have not considered any change in $R$, the second has not been accounted for.

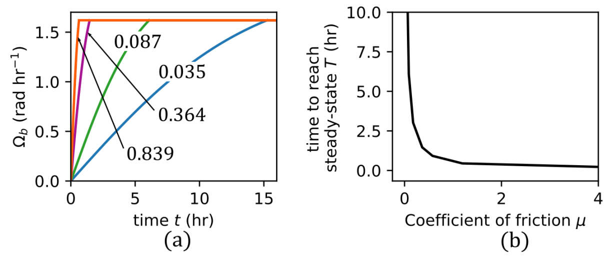


**Figure 3: (a) Evolution of $\Omega_b$ with time for various coefficients of friction $\mu$. (b) Variation of the time taken to reach steady state *T* with $\mu$. $\Omega_i = 1.62$ rad/hr for all cases.**

Our simple demonstration predicts the fate of regolith (here, the block) that is likely to be shed after undergoing surface motion. At low initial spin rates i.e. for $\Omega_i < \Omega_i^{max}$, regolith equilibrates in the frame of rotation of the core. For initial spin rates greater than $\Omega_i^{max}$, regolith gradually loses contact with the core as it spins up to $\Omega_o$, but never leaves the surface i.e. remains *loosely bound* to the core as mentioned for Bennu in [5]. Thus, theoretically, *mass shedding never occurs due to landslides.* The regolith asymptotically enters orbit around the core.

When computational studies are performed, discontinuities on the surface or temporal and spatial discretisation and the associated numerical errors may cause the normal reaction $N$ to become exactly zero or even positive, resulting in *negligible* changes in the spin rate *specifically* due to mass ejection [3]. However, the solutions presented here are free of such errors due to their analytical nature and hence offer a *conceptual understanding of the equilibrium state* resulting from such phenomena.

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**References:** [1] Dobrovolksis A. R. and Burns J. A. (1984) *Icarus, 57,* 464-476. [2] Scheeres D. J. et al. (2020) *JGR: Planets, 125,* e2019JE006284. [3] Brack D. and McMahon J. (2019) *Icarus, 333,* 96-112. [4] Hirabayashi M. et al. (2020) *Icarus, 352*, 113946. [5] Scheeres D. J. et al. (2020) *Sci. Adv., 6(41),* eabc3350.