Equilibrium landscape of top-shaped asteroids

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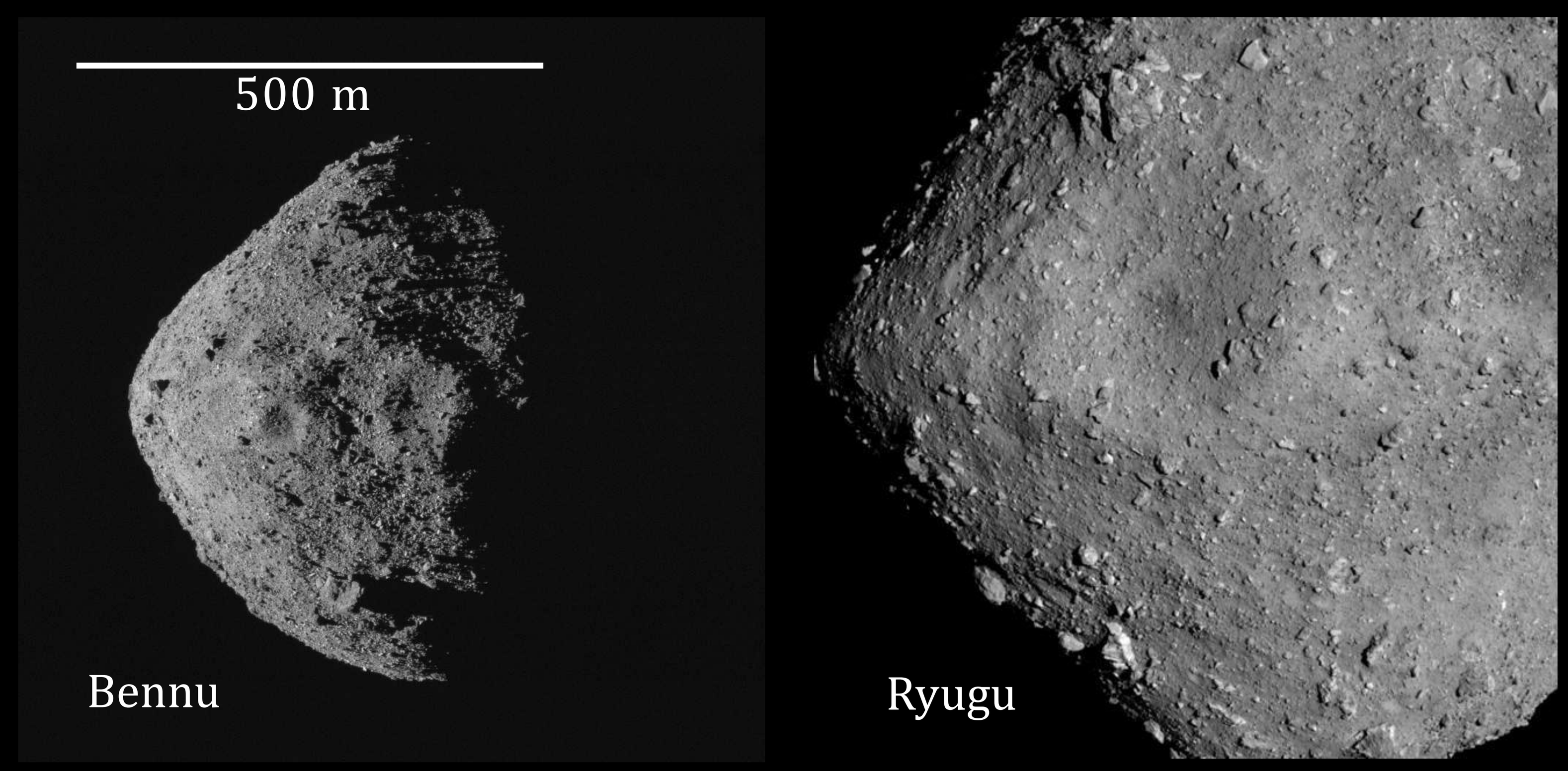


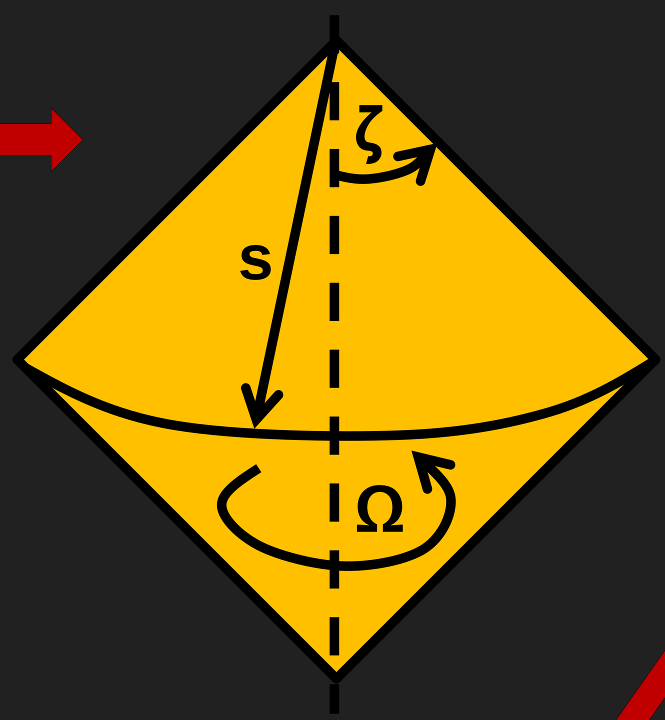
Introduction:
First ever mission to orbit a minor planet - NEAR (Near Earth Asteroid Rendezvous) by NASA in 1996

Recent missions - touch down on the surface of asteroids to collect loose material or *regolith* –
- OSIRIS-REx by NASA (2020) – asteroid 101955 Bennu
- Hayabusa-2 by JAXA (2019) - asteroid 162173 Ryugu

These two belong to the class of asteroids which are called *rubble-piles* described to be an aggregation of loose granular material instead of being a single solid chunk of rock. Additionally, they are *top-shaped*. Bennu, amongst all, has been recently observed to be *active* in nature because of mass shedding events captured by the OSIRIS-REx camera.

While primitive shapes like spheres, spheroids, and triaxial ellipsoids have been studied, the *top shape* has not been.

Here, we investigate the equilibrium landscape of the latter and understand the dynamics of a single grain on the surface, which can to an extent justify the *mass shedding* events that have been observed on Bennu. An imaginary top is constructed by joining two cones base to base.



Gravity field of a double-cone top: The gravitational *potential* for a circular lamina is standard and is available in [1]. We numerically integrate the same along the height of the cone. For a disc of radius a at a height of z from the center of the cone, the formula for gravitational potential $\varphi_{disc}(\boldsymbol{r},a)$ at any point with position vector $\boldsymbol{r}(=R\boldsymbol{e_r}+Z\boldsymbol{e_k})$ not on the disc is given as,

$$\varphi_{disc}(\boldsymbol{r},a) = 2G\rho_o[-\pi|\chi_p|\varepsilon_p + \delta_p\boldsymbol{E}(k) + \frac{a^2-r^2}{\delta_p}\boldsymbol{K}(k) + \frac{\chi_p^2}{\delta_p}\frac{a-R}{a+R}\boldsymbol{\mu}(m^2,k)]\,dz$$

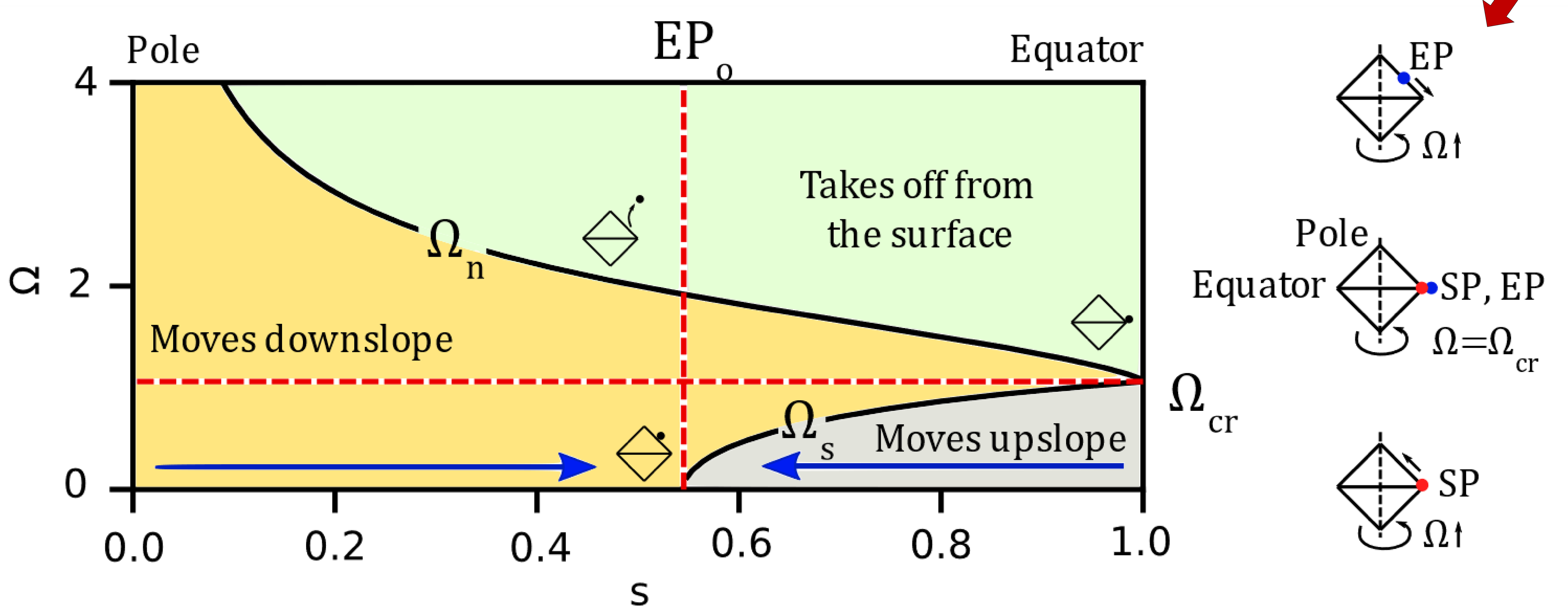
Where, G is the universal gravitational constant, ρ_o is the density, $\chi_p = Z - z$, $\delta_p^2 = (a+R)^2 + \chi_p^2$, $k = \frac{2\sqrt{aR}}{\delta_p}$, $m = \frac{2\sqrt{aR}}{a+R}$, dz is the differential thickness of the disc and $\varepsilon_p = (1,0.5,0)$ for $(R < a, R > a, R = a)$ respectively. Here, $\boldsymbol{E}(k)$, $\boldsymbol{K}(k)$ and $\boldsymbol{\mu}(m^2,k)$ are the elliptic integrals of the first, second and third kinds. For details refer to [2].

This is integrated along the height of the double-cone top to obtain it's gravitational potential. Finally, the gradient of this is calculated using a finite difference scheme to obtain the gravitational acceleration.

The *effective gravity* (for a stationary grain) is given by,

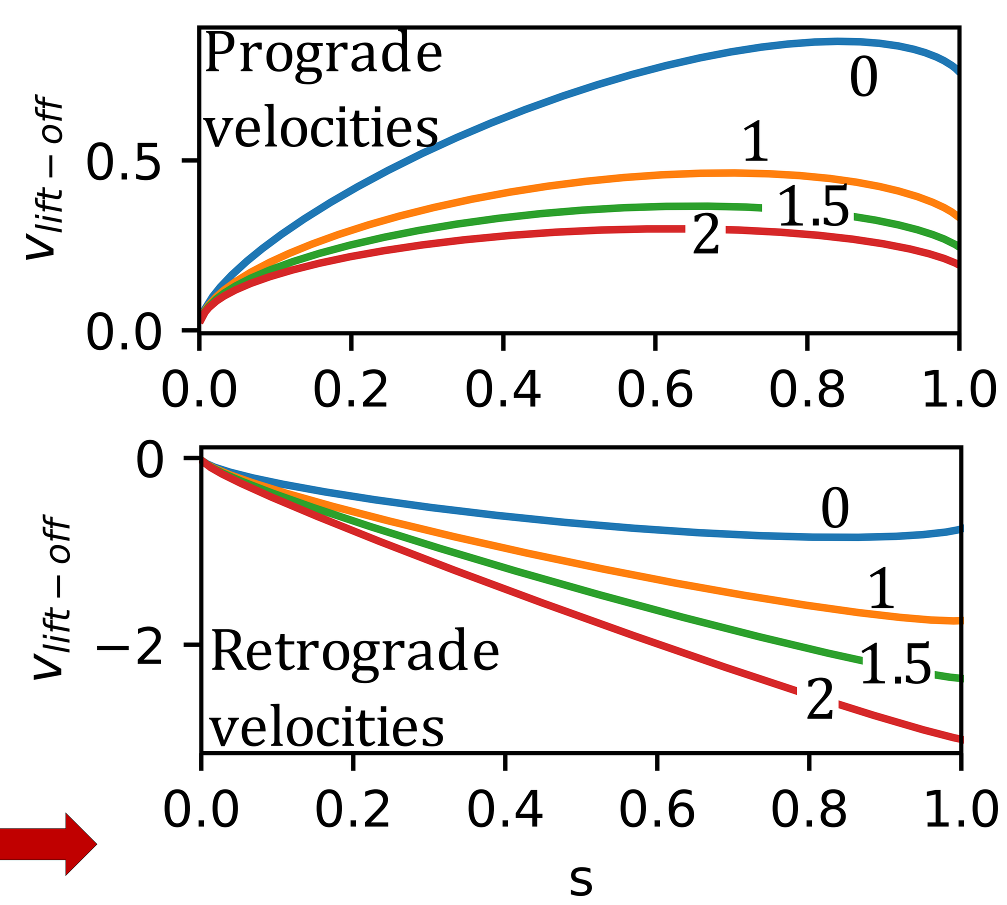
$$\underset{\text{Centrifugal}}{A = \Omega\times(\Omega\times\boldsymbol{r})} - \underset{\text{Gravity}}{G(s)} = \underset{\text{Downslope}}{a_s(s)\boldsymbol{e_s}} + \underset{\text{Normal}}{a_n(s)\boldsymbol{e_n}}$$

This expression has been appropriately nondimensionalized by the dominant scale of each physical quantity.



Results and discussion (Equilibrium Landscape): At a given downslope location s, the spin rate at which a_s and a_n are zero called the threshold spin rate for equilibration $\Omega_s(s)$ and shedding $\Omega_n(s)$ respectively. Alternatively, the downslope location where a_s and a_n vanishes at a particular spin rate are labelled as the *equilibration point* (EP) and the *shedding point* (SP), respectively. The figure above shows the variation of $\Omega_s(s)$ and $\Omega_n(s)$ with s for semi-apex angle $\zeta = 45^o$. The different colors represent the different nature of motion of a grain on or off the surface of the top.

For zero spin rate, the EP *lies at* $s = 0.573$, *which is labelled as* EP_o. As the spin rate of the core is raised, the EP moves downslope. The EP *reaches the equator* when $\Omega = \Omega_s(s=1) = \Omega_{cr}$, which is also the rate at which *mass shedding initiates from the equator* i.e., $\Omega_{cr} = \Omega_n(s=1)$. *In this case* Ω_{cr}*=1.069.* As the spin rate is further elevated, the SP moves upslope and *reaches the pole* $(s=0)$ asymptotically at infinite spin rate. In other words, shedding initiates from higher latitudes when Ω is increased.



Lift-off velocities: For mobile grains, the normal force on them is given by,

$$\cos\zeta\frac{v^2}{\tau} + 2v\Omega\cos\zeta + a_n,$$

where, v is the *azimuthal velocity*, the first term is due to the azimuthal *curvature* of the cone, the second is the *Coriolis acceleration* and the third is the *effective gravity*, all in the *surface normal* direction.

The *lift-off velocities* are determined by setting the above to zero and solving for v. In the figure, prograde (in the direction of rotation of the top, CCW) and retrograde (CW) lift-off velocities are shown for different spin rates. Both *decrease* with a rise in spin rate. However, *the former has a smaller magnitude compared to the latter*. This is because the Coriolis acceleration acts away from the axis of rotation for prograde motion, it acts inwards for retrograde motion.

Reference: 1. Jean-Marc Hure. A key-formula to compute the gravitational potential of inhomogeneous discs in cylindrical coordinates. CMDA. 2012.
2. D. Banik. Spin evolution of top-shaped asteroids due to landslides. Masters Thesis. Indian Institute of Technology Kanpur 2021.